ANNAI VIOLET ARTS AND SCIENCE COLLEGE

DEPARTMENT OF MATHEMATICS

CONTINUOUS INTERNAL ASSESSMENT – II (ODD SEM.)

SUBJECT :ALGEBRA

Class :I B.Sc. Mathematics **Date :1.11.22**

Max.Marks :75 **Sub. Code: SM21A**

PART A (10× 2 = 20 Marks)

Answer any TEN questions

1. Frame an equation with rational coefficients, one of whose roots is $\sqrt{5}+\sqrt{2}$.
2. Solve the equation $x^4-5x^3+4x^2+8x-8 = 0$ given that one of the roots is $1-\sqrt{5}$.
3. Define Symmetric function of the roots.
4. If α,β,γ are the roots of the equation $x^3+ax^2+bx+c = 0$, from the equation whose roots are $\alpha\beta,\beta\gamma$ and $\alpha\gamma$.
5. Define Hermition matrix
6. Given an example of 3x3 symmetric matrix
7. Find the sum of all divisors of 180.
8. Find the product of all divisors of 120.
9. Find $\emptyset(300)$
10. Define Euler's function $\emptyset(N)$.
11. Define a orthogonal matrix.
12. Determine A is $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}A = \begin{pmatrix} 4 & -6 \\ 2 & 1 \end{pmatrix}$.

PART B – (5× 5 = 25 Marks)

Answer any FIVE questions

13. If α,β,γ be the roots of the biquadratic equation $x^4+px^3+qx^2+rx+s = 0$. Find i) $\sum\alpha^2$, ii) $\sum\alpha^2\beta\gamma$, iii) $\sum\alpha^2\beta^2$ iv) $\sum\alpha^3\beta$ v) $\sum\alpha^4$
14. Solve the equation $81x^3-18x^2-36x+8 = 0$ whose roots are in harmonic progression.
15. Solve the equation $x^4-2x^3+4x^2+6x-21= 0$. Given that two of its roots are equal in magnitude and opposite in sign.
16. Verify Cayley Hamilton Theorem for the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$
17. Find the characteristic roots of the matrix $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$
18. Prove that number prime is infinite
19. Find the highest of 3 dividing 1000!

PART C – (3× 10 = 30 Marks)

Answer ANY THREE questions

20. Show that the roots of the equation $x^3+px^2+qx+r = 0$ are in arithmetic progression if $2p^3-9pq+27 = 0$ show that the above condition is satisfied by the equation. $x^3-6x^2+13x-10 = 0$. Hence solve the equation.
21. Find the condition that the roots of the equation $ax^3+3bx^2+3cx+d = 0$ may be in geometric progression.
22. Verify that the matrix A satisfies the characteristic equation and find A^{-1}. $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$
23. a) Find the numbers of the integers which are less than 500 and prime to it.
    b) Find the sum of all integers which are less than 500 and prime to it.
24. Check whether it is diagonalizable or not? $A= \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$